DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MIDTERM MATH 765 - Fall 2010

Your Name: ______________________________

1	(50)	________
2	(50)	________
3	(75)	________
4	(75)	________
BONUS	(50)	________
Total	(250)	________

(1) (50 points)
Show that
$$1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

(2) (50 points) Show that the function $f(x) = sin(1/x)$ is continuous, but not uniformly continuous on $(0, 2\pi]$.

(3) (75 points)
Let $a_0 = 1, a_2 = 2$. Prove that the sequence defined by

$$a_{n+2} = \frac{a_n + a_{n+1}}{2}, n \geq 0$$

is convergent.

(4) (75 points) Let A be a non-empty subset of $\mathbf{R}^1$. Define the function

$$f_A(x) := \inf\{|x-a| : a \in A\}.$$

Prove that f_A is uniformly continuous on $\mathbf{R}^1$.

(5) (**Bonus problem 50 points**) NO PARTIAL CREDIT ON THE BONUS PROBLEM, I.E. ONLY FULL CREDIT OR NO CREDIT.
Let x_n be a sequence of real numbers. Assuming that

$$\lim_n (2x_{n+1} - x_n) = x,$$

show that $\lim_n x_n = x$.