DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MIDTERM MATH 765 - Fall 2007

Your Name: ______________________________

1	(75)	________
2	(75)	________
3	(75)	________
4	(75)	________
BONUS	(50)	________
Total	(300)	________

(1) Let $x_0 :\geq 2$ and
$$x_{n+1} = 1 + \sqrt{x_n - 1}.$$
Show that the sequence $\{x_n\}$ converges and compute its limit.
Hint: First, show by induction that $x_n \geq 2$ and $x_{n+1} \leq x_n$.

(2) Let $\{x_n\}$ be a Cauchy sequence. Assume that a given subsequence $\{x_{n_k}\}$ is convergent and $\lim_{k\to\infty} x_{n_k} = a$. Show that $\{x_n\}$ itself is convergent and $\lim_{n\to\infty} x_n = a$

(3) Let $f(x), g(x), h(x) : (a, b) \to \mathbf{R}^1$ be three functions, so that

$$f(x) \leq g(x) \leq h(x).$$

Assume $f(x), h(x)$ are continuous at a point $c \in (a, b)$ and $f(c) = g(c) = h(c)$. Prove that g is continuous at c as well.

(4) For which values of $\alpha > 0$ is the function

$$f_\alpha(x) = \begin{cases} |x|^\alpha \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

differentiable?

(5) (**Bonus problem 50 points**) NO PARTIAL CREDIT ON THE BONUS PROBLEM, I.E. ONLY FULL CREDIT OR NO CREDIT.

Is the function $f(x) = \ln^2(x) : (1, \infty) \to \mathbf{R}^1$ uniformly continuous? Justify your answer.