

**DEPARTMENT OF MATHEMATICS, UNIVERSITY OF KANSAS**  
**MATH 121, SPRING 2011, MIDTERM EXAM**

Your Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

On this exam, you may use a calculator, but no books or notes. It is not sufficient to just write down the answers. You must explain how you arrived at your answers and how you know they are correct.

Problem	Points	SCORE
1 - 6	60	
7	20	
8	20	
9	20	
10	20	
11	20	
12	20	
13	20	
<b>TOTAL</b>	<b>200</b>	

## Part A - Multiple Choice Examination

Each right answer is worth 10 points

(60 points) *Select only one answer for each problem.*

1. The function

$$f(x) = \ln(x) + \sqrt{2x - 1}$$

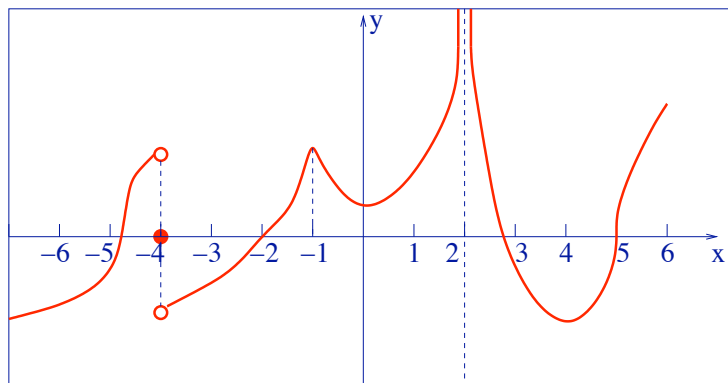
is continuous on

- (A)  $(0, 1/2)$       (B)  $(-1, 1/2)$       (C)  $[1/2, \infty)$   
 (D)  $[-1/2, \infty)$       (E)  $[-1, \infty)$       (F) None of the above is necessarily true

2. Suppose that the function  $f(x)$  is continuous on  $[-1, 1]$  and  $f(-1) = 1$ ,  $f(1) = -1$ . Which of the following must be true?

- (A)  $f(0) = 0$ .  
 (B)  $f(x)$  is decreasing.  
 (C) There exists a  $c$  so that  $-1 < c < 0$  and  $f(c) = 0$ .  
 (D) There exists a  $c$  so that  $-1 < c < 1$  and  $f(c) = -3/2$ .  
 (E) There exists a  $c$  so that  $-1 < c < 1$  and  $f'(c) = 0$ .  
 (F) None of the above is true.

3. Based on your observation only, decide whether each of the following statements is true or false for the function  $y = f(x)$  graphed below.



T F  $f$  is differentiable at the point  $x = -4$ .

T F  $f$  is continuous at the point  $x = -1$ .

T F  $f$  is differentiable at the point  $x = 2$ .

T F  $f$  is continuous at the point  $x = 2$ .

4. Circle the correct statement. At  $x = 0$ , the function given by

$$f(x) = \begin{cases} \sin x & \text{when } x \leq 0, \\ x & \text{when } x > 0 \end{cases}$$

- (A) undefined
- (B) continuous but not differentiable.
- (C) differentiable but not continuous.
- (D) neither continuous nor differentiable.
- (F) both continuous and differentiable.

5. For the function  $f(x) = \ln(1 + x^2)$ , which limit represents  $f'(0)$ :

- (A)  $\lim_{h \rightarrow 0} \frac{\ln(1 + h^2)}{h}$
- (B)  $\lim_{h \rightarrow 0} \frac{\ln(1 + (x + h)^2) - \ln(1 + x^2)}{h}$
- (C)  $\lim_{h \rightarrow 0} \frac{\ln(1 + (x + h)^2) - \ln(1 + x^2)}{x}$
- (D)  $\lim_{h \rightarrow 0} \frac{\ln(1) + [\ln(h)]^2}{h}$
- (F) None of the above.

6. Let  $f$  be a differentiable function, so that  $f(1) = 2$ , and  $f'(1) = -5$ . Let

$$g(x) = \frac{x^3 f(x)}{e^x + 1}$$

Find  $g'(1)$ .

- (A)  $e - 1$
- (B)  $\frac{1 - e}{(1 + e)^2}$
- (C)  $\frac{1 - e}{(1 - e)^2}$
- (D)  $\frac{1 + e}{(1 + e)^2}$
- (F) None of the above.

**Part B - Essay questions**

7. (20 points) Let  $c$  be a constant and  $f(x) = \begin{cases} 2x^2 & \text{if } x \leq c \\ x + 1 & \text{if } x > c \end{cases}$ .

a) Evaluate  $\lim_{x \rightarrow c^-} f(x)$ .

b) Evaluate  $\lim_{x \rightarrow c^+} f(x)$ .

c) For what values of  $c$  is  $f(x)$  continuous?

8. (20 points) Find the equation of the tangent line to the curve

$$e^{xy} = x - y$$

at the point  $(1, 0)$ .

9. (20 points) A particle moves along the path described by the parametric equations

$$x = 2 + 2 \cos t \quad y = 3 \sin t \quad -\pi \leq t \leq 0$$

a. Eliminate the parameter to get a Cartesian equation in  $x$  and  $y$ .

b. Sketch the graph below. Label the points corresponding to  $t = -\pi$ ,  $t = -\pi/2$  and  $t = 0$  and indicate with an arrow the path the particle traces out.

t	x	y
$-\pi$		
$-\pi/2$		
0		

**10. (20 points)** Galileo discovered that the height of an object tossed vertically in the air near the earth's surface at time  $t$  is given by the formula

$$s(t) = s_0 + v_0t - \frac{1}{2}gt^2,$$

where  $s_0$  is the initial height at time  $t = 0$  and  $v_0$  is the initial velocity at time  $t = 0$ . If a stone is shot with a slingshot vertically upward with an initial velocity of 50 m/s from an initial height of 10 m, find the velocity at time  $t = 2$  and at  $t = 7$ . Explain the change in sign. What is the stone's maximum height and when does it reach that height? Use that the gravitational constant is  $g = 9.8$ .

11. (20 points) Compute each of the following limits exactly or state DNE:

a.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+5} - \sqrt{5}}{x}$

b.  $\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + x}$

c.  $\lim_{x \rightarrow \infty} \frac{2 - x^2}{\pi x^2 - 3x + 1}$

d.  $\lim_{x \rightarrow 0} \frac{(4+x)^2 - 16}{x}$

**12. (20 points)** Find the derivative of the function by using logarithmic differentiation  $f(x) = x^{\ln(x)}$ .

**13. (20 points)** A woodcarver has a cube of wood 10 cm to a side. If he shaves .25 cm off each face, use differentials to estimate the volume of the shavings.