MATHEMATICS 765, Fall 2016

Midterm Exam

- **Problem 1 - 75 pts**
  Prove that the following formula holds for all $n \in N$.

$$\sum_{k=1}^{n}(2k-1)^2 = \frac{n(4n^2-1)}{3}$$

- **Problem 2 - 75 pts**
  Suppose that $x_0 = 1, y_0 = 0$ and define

$$x_n = x_{n-1} + 2y_{n-1},\ y_n = x_{n-1} + y_{n-1}$$

for $n \in N$. Prove that $x_n^2 - 2y_n^2 = \pm 1$ for $n \in N$ and that

$$\lim_{n\to\infty} \frac{x_n}{y_n} = \sqrt{2}.$$

- **Problem 3 - 75 pts**
  Suppose that $\{x_n\}$ is a real sequence. Prove the following identity for the limits supremum and infimum:

$$-\lim_{n\to\infty} \sup x_n = \lim_{n\to\infty} \inf(-x_n)$$

- **Problem 4 - 75 pts**
  Prove that if f, g are uniformly continuous on an interval $[a, b]$ and $g(x) \neq 0$ for $x \in [a, b]$, then the function $\frac{f}{g}$ is uniformly continuous on $[a, b]$.

- **Bonus Problem - 50 pts - full credit or no credit only**
  Suppose that $f : N \to R$. If

$$\lim_{n\to\infty} f(n+1) - f(n) = L,$$

prove that

$$\lim_{n\to\infty} \frac{f(n)}{n}$$

exists and equals L.