

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF KANSAS
MATH 121, SPRING 2011, MIDTERM EXAM

Your Name: _____

Instructor: _____

On this exam, you may use a calculator, but no books or notes. It is not sufficient to just write down the answers. You must explain how you arrived at your answers and how you know they are correct.

Problem	Points	SCORE
1 - 5	40	
6	20	
7	20	
8	20	
9	20	
10	20	
11	20	
12	20	
13	20	
TOTAL	200	

Part A - Multiple Choice Examination

Each right answer is worth 8 points

(40 points) *Select only one answer for each problem.*

1. The function

$$f(x) = \ln(x) + \sqrt{2x - 1}$$

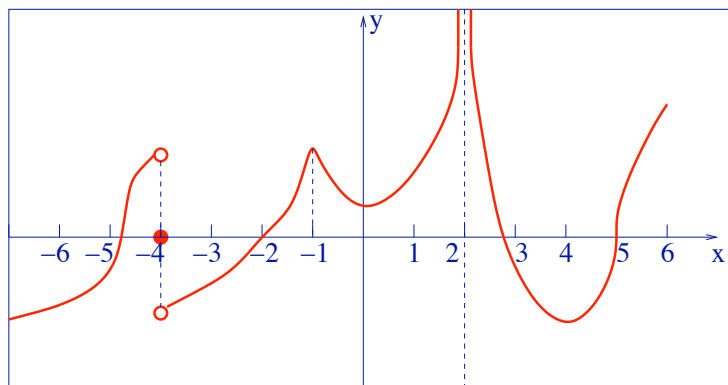
is continuous on

- (A) $(0, 1/2)$ (B) $(-1, 1/2)$ (C) $(1/2, \infty)$
 (D) $[-1/2, \infty)$ (E) $[-1, \infty)$ (F) None of the above is necessarily true

2. Suppose that the function $f(x)$ is continuous on $[-1, 1]$ and $f(-1) = 1$, $f(1) = -1$. Which of the following must be true?

- (A) $f(0) = 0$.
 (B) $f(x)$ is decreasing.
 (C) There exists a c so that $-1 < c < 0$ and $f(c) = 0$.
 (D) There exists a c so that $-1 < c < 1$ and $f(c) = -1/2$.
 (E) There exists a c so that $-1 < c < 1$ and $f'(c) = 0$.
 (F) None of the above is true.

3. Based on your observation only, decide whether each of the following statements is true or false for the function $y = f(x)$ graphed below.



T F (A) f is differentiable at the point $x = -4$.

T F (B) f is continuous at the point $x = -1$.

T F (C) f is differentiable at the point $x = -1$.

T F (D) f is continuous at the point $x = 2$.

4. Circle the correct statement. At $x = 0$, the function given by

$$f(x) = \begin{cases} \sin x & \text{when } x \leq 0, \\ x & \text{when } x > 0 \end{cases}$$

- (A) undefined
- (B) continuous but not differentiable.
- (C) differentiable but not continuous.
- (D) neither continuous nor differentiable.
- (F) both continuous and differentiable.

5. For the function $f(x) = \ln(1 + x^2)$, which limit represents $f'(0)$:

- (A) $\lim_{h \rightarrow 0} \frac{\ln(1 + h^2)}{h}$
- (B) $\lim_{h \rightarrow 0} \frac{\ln(1 + (x + h)^2) - \ln(1 + x^2)}{h}$
- (C) $\lim_{h \rightarrow 0} \frac{\ln(1 + (x + h)^2) - \ln(1 + x^2)}{x}$
- (D) $\lim_{h \rightarrow 0} \frac{\ln(1) + [\ln(h)]^2}{h}$
- (F) None of the above.

Part B - Essay questions

6. (20 points) Show that the function

$$f(x) = \frac{1 - e^x}{1 + e^x}$$

is invertible and find the inverse f^{-1} . Find the domain and the range of f, f^{-1} .

7. (20 points) Let the curve γ be given by the parametric equations:

$$\begin{cases} x(t) = \sin^2(t) \\ y(t) = \cos(t) \end{cases}$$

Find the Cartesian equations of this curve, graph it and describe the trajectory and direction in which a point on it moves, if $-\pi/2 \leq t \leq \pi/2$.

8. (20 points) Use the intermediate value theorem to show that the equation

$$\tan^{-1} x = 1 - x$$

has a solution in $(0, 1)$.

- 9. (20 points)** For the function $G(x) = x(x - 1)(x + 2)$, find the equation of the tangent line at $P(1, 0)$. Determine the intervals, where the function is
- increasing, decreasing,
 - concave up, concave down.

10. (20 points) Compute the limit

$$\lim_{x \rightarrow 1} \left(\frac{1}{x-1} + \frac{1}{x^2 - 3x + 2} \right)$$

11. (20 points) For the function

$$f(x) = \begin{cases} x + 2 & x < 0 \\ e^x & 0 \leq x \leq 1 \\ 2 - x & 1 < x \end{cases}$$

find the points where the function is

- a) not continuous,
- b) not differentiable.

Provide justification of your claims.

12. (20 points) A woodcarver has a cube of wood 20 cm to a side. If he shaves .25 cm off each face, use differentials to estimate the volume of the shavings.

13. (20 points) Let f be a differentiable function, so that $f(1) = 2$, and $f'(1) = -5$. Let

$$g(x) = \frac{x^3 f(x)}{e^x + 1}$$

Find $g'(1)$.