DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 121, SPRING 2014
SAMPLE FINAL EXAM

Your Name: ________________________ **Instructor:** ________________________

Problem	Points	SCORE
1 - 10	100	
11	25	
12	25	
13	25	
14	25	
15	25	
16	25	
17	25	
18	25	
TOTAL	300	

Part A - Multiple Choice Examination

Each right answer is worth 10 points. Select only **one** answer for each problem.

1. Suppose $f'(x) = e^{-x^2}$. Which of the following is NOT necessarily true?

   (a) f is continuous on $(-\infty, \infty)$
   (b) f is increasing
   (c) f is concave down on $(0, \infty)$
   (d) f is decreasing
   (e) f' is concave up on $(1, \infty)$

2. At what point on the curve $y = \ln x$ is the tangent line parallel to the line $x - 4y = 1$.

   (a) $(2, \ln 2)$
   (b) $(4, \ln 4)$
   (c) $(4, \frac{3}{4})$
   (d) $(2, \frac{1}{2})$
   (e) $(1, 0)$

3. Which integral matches the Riemann sum?

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(\frac{4n^2+i^2}{n^2}\right)^2$$

(a) $\int_0^1 x\,dx$ (b) $\int_2^3 (4x+1)^2\,dx$ (c) $\int_0^1 \frac{dx}{x^3}$ (d) $\int_0^1 (4+x^2)^2\,dx$

4. The value of the integral $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ is

(a) 2π (b) 5π (c) 2 (d) 24 (e) 3 (f) none of the above.

5. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of $6cm/sec$. Find the rate at which the area within the circle is increasing after 3 seconds.

(a) 212π (b) 5π (c) 216π (d) 234π (e) $34\pi^2$ (f) none of the above.

6. Which of the following is the correct form of the partial fraction expansion of

$$f(x) = \frac{3x^2 - 2x + 7}{(x-1)(x^2+1)}.$$

(a) $\frac{A}{x-1} + \frac{B}{x^2+1}$

(b) $\frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$

(c) $\frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

(d) $\frac{A}{x-1} + \frac{Bx+C}{(x-1)^2} + \frac{D}{x+1}$

(e) $\frac{A}{x-1} + \frac{B}{x-1} + \frac{C}{x+1}$

7. $\lim_{h \to 0} \frac{1}{h} \ln \left(\frac{3+h}{3} \right) =$

(a) $\frac{1}{3}$ (b) 1 (c) e^3 (d) $\ln 3$ (e) does not exist.

8. Which of the following statements about the function $f(x) = x^4 - 2x^3$ is true?

(a) The function has no relative extremum

(b) The graph of the function has one point of inflection and two relative extrema

(c) The graph of the function has two points of inflection and one relative extremum

(d) The graph of the function has two points of inflection and two relative extrema

(e) The graph of the function has two points of inflection and three relative extrema

9. The arc length of the parametric curve given by $x(t) = \cos(t)$ and $y(t) = \sin(t)$ for t in the interval $[0, 1]$ is

(a) π (b) 1 (c) 2π (d) $\pi/2$ (e) 2.

10. The value of the definite integral $\int_0^1 (x^2 + 1)e^{-x} dx$ is:

(a) $-\frac{5}{e} + 2$ (b) $1 + \frac{1}{e}$ (c) $2 - \frac{2}{e}$ (d) $e/2$ (e)$3 - \frac{6}{e}$

Part B - Show your work to get full credit.

Each problem is worth 25 points.

11. Find an equation of the tangent line at the point $(0, 0)$ for the function $y = \frac{2x}{(x+1)^2}$.

12. Use logarithmic differentiation to find the derivative of the function

$$y = (2x + 1)^5(x^4 - 3)^6.$$

13. Find the limits

$$\lim_{x\to\infty}(\sqrt{x-1}-\sqrt{x+1})$$

$$\lim_{x\to 0^+}(\frac{1}{x}-\frac{1}{x^2+x})$$

$$\lim_{x\to 0} x^2\sin(\frac{1}{x})$$

14. You want to paint one side of a fence that starts 1m tall and whose height in meters is given by

$$h(x)=\frac{1}{(x+1)^2},$$

where x is the distance in meters from the beginning of the fence. If you don't know how long the fence is, what is the area that you should be prepared to paint and still be guaranteed to be able to paint all of the fence. Explain your solution.

15. Evaluate the integral

$$\int_1^4 x^{3/2} \ln x dx$$

16. Evaluate the integral

$$\int \frac{dt}{\cos^2 t\sqrt{1 + \tan t}}$$

17. Consider the region R in the first quadrant bounded by $y = x^2$ and $y = \sqrt{x}$. Compute the volume of the solid obtained by rotating the region R about the y-axis.

18. Evaluate the improper integral or otherwise show it is divergent

$$\int_1^\infty \frac{\ln x}{x} dx$$