

MATHEMATICS 647, SPRING 2016

Final Exam

- **Problem 1 - 80 pts**

Determine the eigenvalues and eigenvectors of the following Sturm-Liouville problem.

$$y'' + \lambda y = 0, \quad y(0) + y'(0) = 0, \quad y(1) + y'(1) = 0.$$

- **Problem 2 - 80 pts**

A cantilevered beam is fixed at one end and free at the other. Use separation of variables to describe the motion of such beam of length L , which is modeled by the initial boundary value problem: $u_{tt} = -c^2 u_{xxxx}$

$$u(0, t) = 0, \quad u_x(0, t) = 0, \quad u_{xx}(L, t) = 0, \quad u_{xxx}(L, t) = 0$$

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x), \quad 0 < x < L.$$

- **Problem 3 - 80 pts**

Use the Fourier transform method to solve the given problem for $-\infty < x < \infty$, $t > 0$:

$$u_t = e^{-t} u_{xx}, \quad u(x, 0) = e^{-|x|}.$$

- **Problem 4 - 80 pts**

Use convolution and Fourier transform to solve the problem for $-\infty < x < \infty$, $t > 0$:

$$u_t = c^2 u_{xx} + k u_x, \quad k > 0, \quad u(x, 0) = f(x).$$

- **Problem 5 - 80 pts**

Find the Fourier transform of the piece-wise constant function plotted below:

