# MATH 950, FALL 2015

# Homework 1 - due Monday, September 14

- **Problem 1 (5 points)**
  For the sine-Gordon equation $u_{tt} = u_{xx} - \sin u$ define traveling kink solutions as $u(x,t) = f(x-ct)$ such that $f(z) \to 0$ as $z \to -\infty$ and $f(z) \to 2\pi$ as $z \to +\infty$. Describe the equation that the profile function $f(z)$ satisfies and the speeds $c$, for which traveling kinks exist. Show that the profile function $f(z) = 4\arctan[\exp(\frac{z}{\sqrt{1-c^2}}]$ solves this equation and sketch the resulting kink waves.
- **Problem 2 (5 points)**
  Find the solution of $(x+1)^2 u_x + (y-1)^2 u_y = (x+y)u$ satisfying the condition $u(x,0) = -1-x$ for $-1 < x < \infty$. Where in the $xy$-plane is $u(x,y)$ determined by these conditions?
- **Problem 3 (5 points)**
  Show that all the projected characteristic curves of
  $$(2x+u)u_x + (2y+u)u_y = u$$
  through the point $(1,1)$ are given by the straight line $y = x$.
- **Problem 4 (5 points)**
  Solve $xu_x + yu_y + (u_x^2 + u_y^2)/2 = u$ with initial condition $u(x,0) = (1-x^2)/2$.
- **Problem 5 (5 points)**
  Solve the equation $(u_x)^2 + (u_y)^2 = 1$ with initial data given by $s \to (\sin s, \cos s, 0)$ for $0 \le s \le \pi/2$. Based on the method of characteristics where in the $xy$-plane is $u(x,y)$ determined by these conditions?
- **Problem 6 (5 points)**
  Read the application to geometrical optics section on p.$36-40$ in McOwen's book and then work out problem 9 on page 42.