Your Name: _____

On this exam, you may use a calculator and a page of notes.

It is not sufficient to just write down the answers. You must explain how you arrived at your answers and how you know they are correct.

1	(100)	_____
2	(100)	_____
3	(100)	_____
4	(100)	_____
Total	(400)	_____

(1) Consider the linear transport equation
$$u_t + (1 + x^2)u_x = 0$$
Find and sketch the characteristic curves, write a formula for the general solutions and find the solution to the initial value problem $u(x, 0) = 1 - e^{-x^2}$

(2) Use D'Alembert's formula to solve the one dimensional wave equation
$$u_{tt} = 4u_{xx},$$
subject to initial conditions
$$u(x, 0) = e^{-x^2}, \frac{\partial u}{\partial t}(x, 0) = \sin x$$

(3) Solve problem 4.2.9 from the book by P. Olver.

(4) Solve the non-homogeneous boundary value problem for the one dimensional heat equation on a bar with unit length and $c = 1$, for the following data:
$$u(0, t) = 100, u(1, t) = 50, u(x, 0) = x.$$