

**DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 220 - FALL 2010 - EXAM 1**

Your Name: _____

On this exam, you may use a calculator and one page with formulas. It is not sufficient to just write down the answers. You must show your work to get full credit.

1	(40)	_____
2	(40)	_____
3	(40)	_____
4	(40)	_____
5	(40)	_____
Bonus	(20)	_____
Total	(200)	_____

- **Problem 1**

Solve the initial value problem explicitly and sketch the graph of the solution

$$\left| \begin{array}{l} y' = \frac{-x}{1+y} \\ y(0) = 1 \end{array} \right.$$

What is the domain of the solution?

- **Problem 2**

Solve the equation implicitly

$$(3x^2 + y^2) + (2xy - 6y^2)y' = 0$$

- **Problem 3** Find the solution of the given initial value problem and describe the behavior of the solution for large t .

$$y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 2$$

Describe the behavior of the solution as $t \rightarrow \infty$.

- **Problem 4** Solve the initial-value problem

$$y'' + y' - 2y = 2t, \quad y(0) = 0, \quad y'(0) = 1$$

- **Problem 5**

Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in a rate of 2 L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

- **Extra Credit Problem (20 points)** Solve the initial-value problem with a parameter b .

$$y'' - y' + 0.25y = 0, \quad y(0) = 2, \quad y'(0) = b$$

Find the critical value of b that separates solutions that always remain positive from those that eventually become negative.