MATH 800 - FINAL EXAM Practice Problems

(1) Compute
$$\int_{-\infty}^{\infty} \frac{\cos(2x)}{x^4 + 1} dx.$$

(2) Compute
$$\int_{-\infty}^{\infty} \left(\frac{\sin x}{x} \right)^2 dx.$$

(3) For which values of z, does the following series converge
$$\sum_{n=0}^{\infty} \left(\frac{z^n}{n!} + \frac{n^2}{z^n} \right)$$

(4) Let g_n be a sequence of entire functions, with only real zeros. Suppose that g_n converges uniformly on the compact subsets to a function g. Show that g has only real zeros as well.

(5) How many zeros does the polynomial
$$3z^9 + 8z^6 + z^5 + 2z^3 + 1,$$
has in the annulus $\{z : 1 < |z| < 2\}$?

(6) Evaluate the integrals
$$I_k = \oint_\gamma \frac{1}{z(z-1)\dots(z-k)} dz, \ J_k = \oint_\gamma \frac{(z-1)\dots(z-k)}{z} dz$$
where γ is a simple closed contour containing all $0, 1, \dots, k$.

(7) Suppose that $f : D(0, 1) \to \mathbb{C}$, so that f^2 and f^3 are analytic. Show that f is analytic as well.

(8) Compute the Laurent series at $z = 0$ for the function
$$f(z) = \frac{1}{z(z-1)(z-2)}$$
What is the domain of convergence?

(9) Suppose f is analytic on $D(0, 1)$, so that $|f(z)| \le 1$ for $|z| = 1$ and $f(\alpha) = 0$. Show that
$$|f(z)| \le \left| \frac{z - \alpha}{1 - \bar{\alpha}z} \right|$$